Unit - 1
Partial Fraction

A way of "Breaking apart" Fraction with polinomial in them.

Ex - $\frac{2}{x-2} + \frac{3}{x+1}$

$$\frac{2(x+1) + 3(x-2)}{(x-2)(x+1)} = \frac{2x+2+3x-6}{x^2+x-2x-2}$$

$$= \frac{5x-4}{x^2-x-2}$$

Find Partial Fraction :-

(1) $\frac{5x-4}{x^2-x-2}$

$$\frac{5x-4}{x^2-2x+x-2} \Rightarrow \frac{5x-4}{x(x-2)+1(x-2)}$$

$$\frac{5x-4}{(x-2)(x+1)} = \frac{A_1}{(x-2)} + \frac{A_2}{(x+1)} \quad \text{--- (i)}$$

$$5x-4 = \frac{A_1(x+1) + A_2(x-2)}{(x-2)(x+1)}$$

$$5x-4 = A_1(x+1) + A_2(x-2) \quad \text{--- (ii)}$$

Substituting the root of $(x-2)$ is $x-2=0$

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$x = 2$ in eq^n – (ii)

$5x - 4 = A_1(x+1) + A_2(x-2)$

$5 \times 2 - 4 = A_1(2+1) + A_2(2-2)$

$10 - 4 = A_1(3) + A_2 \times 0$

$6 = 3A_1 + 0$

$6 = 3A_1$

$A_1 = \frac{6}{3}$, $A_1 = 2$

Substitution the root of $(x+1)$ is $(x+1) = 0$

$x = -1$ in eq^n (ii)

$5(-1) - 4 = A_1(-1+1) + A_2(-1-2)$

$-5 - 4 = A_1(0) + A_2(-3)$

$-9 = 0 + A_2(-3)$

$9 = 3A_2$

$A_2 = \frac{9}{3} \Rightarrow A_2 = 3$

Putting value of A_1 and A_2 in eq^n – (1)

$$\frac{5x-4}{(x-2)(x+1)} = \frac{A_1}{(x-2)} + \frac{A_2}{(x+1)}$$

$$\frac{5x-4}{(x-2)(x+1)} = \frac{2}{(x-2)} + \frac{3}{(x+1)}$$

(2) $\frac{x+5}{(x-3)(x+1)}$

$$\frac{x+5}{(x-3)(x+1)} = \frac{A_1}{(x-3)} + \frac{A_2}{(x+1)} \quad \text{—(i)}$$

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$$\frac{x+5}{(x-3)(x+1)} = \frac{A_1(x+1)+A_2(x-3)}{(x-3)(x+1)}$$

$$x+5 = A_1(x-3) + A(x+1) \quad \text{—— (ii)}$$

root of $x-3=0$

$x=3$

Putting three value of $x=3$ in eq (ii)

$$x+5 = A_1(x+1)+A_2(x-3)$$

$$3+5 = A_1(3+1)+A_2(3-3)$$

$$8 = 4A_1 + A_2(0) \quad = \quad 8 = 4A_1 + 0$$

$$\frac{8}{4} = A_1 \quad \Rightarrow \quad A_1 = 2$$

root of $x+1=0$, $x=-1$

putting the value of $x=(-1)$ in eq^n-(ii)

$$x+5 = A_1(x+1) + A_2(x-3)$$

$$-1+5 = A_1(-1+1) + A_2(-1-3)$$

$$4 = A_1(0) + A_2(-4)$$

$$4 = 0 + (-4A_2)$$

$$4 = -4A_2$$

$$A_2 = \frac{4}{-4}$$

$$A_2 = -1$$

Substituting the value of $A_1=2$ and $A_2=-1$

$$\frac{x+5}{(x-3)(x+1)} = \frac{A_1}{(x-3)} + \frac{A_2}{(x+1)}$$

$$\frac{x+5}{(x-3)(x+1)} = \frac{2}{(x-3)} - \frac{1}{(x+1)}$$

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(3) $\frac{8x + 14}{(x+4)(x-2)} = ?$

$\frac{8x + 14}{(x+4)(x-2)} = \frac{A_1}{(x+4)} + \frac{A_2}{(x-2} \quad \text{— (i)}$

$\frac{8x + 14}{(x+4)(x-2)} = \frac{A_1(x-2) + A_2(x+4)}{(x+4)(x-2)}$

$8x + 14 = A_1(x-2) + A_2(x+4) \quad \text{— (ii)}$

root of $x+4=0 \Rightarrow x=-4$ in eq (i)

$8(-4) + 14 = A_1(-4-2) + A_2(-4+4)$

$-32 + 14 = A_1(-6) + A_2(0)$

$-18 = -6A_1 + 0$

$A_1 = \frac{-18}{-6} \Rightarrow A_1 = 3$

root of $x-2=0 \Rightarrow x=2$

$8x + 14 = A_1(x-2) + A_2(x+4)$

$8(2) + 14 = A_1(2-2) + A_2(2+4)$

$16 + 14 = 0 + A_2(6)$

$30 = 6A_2$

$A_2 = \frac{30}{6} \Rightarrow A_2 = 5$

Substituting the value of $A_1 = 3$, $A_2 = 3$ in eq (i)

$\frac{8x + 14}{(x+4)(x-2)} = \frac{A_1}{(x+4)} + \frac{A_2}{(x-2)}$

$\frac{8x + 14}{(x+4)(x-2)} = \frac{3}{(x+4)} + \frac{5}{(x-2)}$

(4) $\frac{12x - 1}{(x^2+x-2)}$

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$$\frac{12x-1}{x^2+2x-x-2} \Rightarrow \frac{12x-1}{x(x+2)-1(x+2)}$$

$$\frac{12x-1}{(x+2)(x-1)}$$

$$\frac{12x-1}{(x+2)(x-1)} \Rightarrow \frac{A_1}{(x+2)} + \frac{A_2}{(x-1)} \quad \text{--- (i)}$$

$$\frac{12x-1}{(x+2)(x-1)} = \frac{A_1(x-1) + A_2(x+2)}{(x+2)(x-1)}$$

$$12x-1 = A_1(x-1) + A_2(x+2) \quad \text{--- (ii)}$$

root of $x+2=0 \Rightarrow x=-2$ in eq ---- (ii)

$$12x-1 = A_1(x-1) + A_2(x+2)$$

$$12(-2)-1 = A_1(-2-1) + A_2(-2+2)$$

$$-24-1 = A_1(-3) + A_2(0)$$

$$-25 = -3A_1$$

$$A_1 = \frac{-25}{-3} = 8.33$$

root of $x-1=0 \Rightarrow x=1$ in eq ---- (ii)

$$12x-1 = A_1(x-1) + A_2(x+2)$$

$$12(1)-1 = A_1(1-1) + A_2(1+2)$$

$$12-1 = A_1(0) + A_2(3)$$

$$11 = 3A_2$$

$$A_2 = \frac{11}{3} = 3.66$$

Substituting the value of $A_1 = 8.33$ and $A_2 = 3.66$

$$\frac{12x-1}{(x+2)(x-1)} = \frac{A_1}{(x+2)} + \frac{A_2}{(x-1)}$$

$$\frac{12x-1}{(x+2)(x-1)} = \frac{8.33}{(x+2)} + \frac{3.66}{(x-1)}$$ Ans.

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(5) $\dfrac{x+26}{x^2+3x-10} = ?$

$\dfrac{x+26}{x^2+5x-2x-10}$

$\dfrac{x+26}{(x+5)-2(x+5)}$

$\dfrac{x+26}{(x+5)(x-2)}$

$\dfrac{x+26}{(x+5)(x-2)} = \dfrac{A_1}{(x+5)} + \dfrac{A_2}{(x-2)}$ — (i)

$\dfrac{x+26}{(x+5)(x-2)} = \dfrac{A_1(x-2)+A_2(x+5)}{(x+5)(x-2)}$

$x+26 = A_1(x-2)+A_2(x+5)$ — (ii)

taking root, $x+5=0$, $x=(-5)$

put $x=-5$ in eq^n (ii)

$x+26 = A_1(x-2)+A_2(x+5)$

$-5+26 = A_1(-5-2)+A_2(-5+5)$

$21 = A_1(-7)+A_2(0)$

$21 = -7A_1$

$A_1 = \dfrac{21}{7} \Rightarrow A_1 = -3$

taking root $x-2=0$, $x=2$

put $x=2$ and $A_1=(-3)$ in eq^n (ii)

$x+26 = A_1(x-2)+A_2(x+5)$

$2+26 = -3(2-2)+A_2(2+5)$

$28 = -3(0) + A_2(7)$

$28 = 0 + 7A_2$

$A_2 = \dfrac{28}{7}$

$A_2 = 4$

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Substitution the value of $A_1 = (-3)$, $A_2 = 4$ in eq^n

$$\frac{x+26}{(x+5)(x-2)} = \frac{A_1}{(x+5)} + \frac{A_2}{(x-2)}$$

$$\frac{x+26}{(x+5)(x-2)} = \frac{-3}{(x+5)} + \frac{4}{(x-2)}$$

$$\frac{x+26}{(x+5)(x-2)} = \frac{4}{(x-2)} - \frac{3}{(x+5)}$$

(6) $\frac{4x-8}{x^2-8x+15}$

$$\frac{4x-8}{x^2-5x-3x+15} \Rightarrow \frac{4x-8}{x(x-5)-3(x-5)} \Rightarrow \frac{4x-8}{(x-5)(x-3)}$$

$$\frac{4x-8}{(x-5)(x-3)} = \frac{A_1}{(x-5)} + \frac{A_2}{(x-3)} \quad \text{--- (i)}$$

$$\frac{4x-8}{(x-5)(x-3)} = \frac{A_1(x-3) + A_2(x-5)}{(x-5)(x-3)}$$

$$4x-8 = A_1(x-3) + A_2(x-5) \quad \text{--- (ii)}$$

taking root $x-5=0$, $x=5$

put $x = 5$ — in eq^n (ii)

$$4x-8 = A_1(x-3) + A_2(x-5)$$

$$4(5)-8 = A_1(5-3) + A_2(5-5)$$

$$20-8 = A_1(2) + A_2(0)$$

$$12 = 2A_1$$

$$A_1 = 12/2 \qquad A_1 = 6$$

taking root $(x-3)=0$, $x=3$

put $x=3$ - in eq^n - (ii)

$$4x-8 = A_1(x-3) + A_2(x-5)$$

$$4(3)-8 = A_1(3-3) + A_2(3-5)$$

$$12 - 8 = A_1(0) + A_2(-2)$$
$$4 = 0 + (-2A_2)$$
$$A_2 = \frac{4}{-2} \qquad A_2 = (-2)$$

Substituting the value of $A_1 = 6$, $A_2 = (-2)$ — in eq^n

$$\frac{4x-8}{(x-5)(x-3)} = \frac{A_1}{(x-5)} + \frac{A_2}{(x-3)}$$
$$\frac{4x-8}{(x-5)(x-3)} = \frac{6}{(x-5)} + \frac{(-2)}{(x-3)}$$
$$\frac{4x-8}{(x-5)(x-3)} = \frac{6}{(x-5)} - \frac{2}{(x-3)}$$

(7) $\dfrac{12x-1}{x^2+x-12}$

$$\frac{12x-1}{x^2+4x-3x-12} \Rightarrow \frac{12x-1}{x(x+4)-3(x+4)}$$
$$\frac{12x-1}{(x+4)(x-3)} = \frac{A_1}{(x+4)} + \frac{A_2}{(x-3)} \quad \text{— (i)}$$
$$\frac{12x-1}{(x+4)(x-3)} = \frac{A_1(x-3) + A_2(x+4)}{(x+4)\ (x-3)}$$
$$12x - 1 = A_1(x-3) + A_2(x+4) \quad \text{— (ii)}$$

taking root $x - 3 = 0 \Rightarrow x = 3$

Put the value of $x = 3$ in eq^n (ii)

$$12x - 1 = A_1(x-3) + A_2(x+4)$$
$$12 \times 3 - 1 = A_1(3-3) + A_2(3+4)$$
$$36 - 1 = A_1(0) + 7A_2$$
$$35 = 7A_2$$
$$A_2 = 5$$

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taking root $x+4=0 \Rightarrow x=-4$
put the value of $x=-4$ in eq^n (ii)

$$12x-1 = A_1(x-3) + A_2(x+4)$$
$$12(-4)-1 = A_1(-4-3) + A_2(-4+4)$$
$$-48-1 = A_1(-7) + A_2(0)$$
$$-49 = A_1(-7)$$
$$A_1 = \frac{49}{7}, \quad A_1 = 7$$

Substituting the value of $A_1 = 7$, $A_2 = 5$ in eq^n ii)

$$\frac{12x-1}{(x+4)(x-3)} = \frac{A_1}{x+4} + \frac{A_2}{x-3}$$
$$\frac{12x-1}{(x+4)(x-3)} = \frac{7}{x+4} + \frac{5}{x-3}$$ Ans/

(8) $\dfrac{2x-1}{x^2+7x+12}$

$$\frac{2x-1}{x^2+4x+3x+12} \Rightarrow \frac{2x-1}{x(x+4)+3(x+4)} \Rightarrow \frac{2x-1}{(x+4)(x+3)}$$
$$\frac{2x-1}{(x+4)(x+3)} = \frac{A_1}{(x+4)} + \frac{A_2}{(x+3)} \quad \text{— (i)}$$
$$\frac{2x-1}{(x+4)(x+3)} = \frac{A_1(x+3)+A_2(x+4)}{(x+4)(x+3)}$$
$$2x-1 = A_1(x+3) + A_2(x+4) \quad \text{— (ii)}$$

taking root Ist $x+4=0 \Rightarrow x=-4$
put up the value of $x=-4$ in eq^n -(ii)

$$2x-1 = A_1(x+3) + A_2(x+4)$$
$$2(-4)-1 = A_1(-4+3) + A_2(-4+4)$$
$$-8-1 = A_1(-1) + A_2(0)$$
$$-9 = -A_1 \qquad A_1 = 9$$

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taking root IInd $x+3=0 \Rightarrow x=-3$
put up the value of $x=-3$ in eqⁿ -(ii)

$$2x-1 = A_1(x+3) + A_2(x+4)$$
$$2(-3)-1 = A_1(-3+3) + A_2(-3+4)$$
$$-5-1 = A_1(0) + A_2(+1)$$
$$-6 = 0 + A_2$$
$$A_2 = -7$$

Substituting the value of $A_1 = 9$, $A_2 = -7$ eqⁿ (i)

$$\frac{2x-1}{(x+4)(x+3)} = \frac{A_1}{x+4} + \frac{A_2}{x+3}$$
$$\frac{2x-1}{(x+4)(x+3)} = \frac{9}{(x+4)} + \frac{(-7)}{(x+3)}$$
$$\frac{2x-1}{(x+4)(x+3)} = \frac{9}{(x+4)} - \frac{7}{(x+3)}$$

(9) $\dfrac{17x-1}{3x^2-8x-3}$

$$\frac{17x-1}{3x^2-9x+1x-3} \Rightarrow \frac{17x-1}{3x(x-3)+1(x-3)}$$
$$\frac{17x-1}{(x-3)(3x+1)} = \frac{A_1}{(x-3)} + \frac{A_2}{(3x+1)} \quad \text{--- (i)}$$
$$\frac{17x-1}{(x-3)(3x+1)} = \frac{A_1(3x+1) + A_2(x-3)}{(x-3)(3x+1)}$$
$$17x-1 = A_1(3x+1) + A_2(x-3) \quad \text{--- (ii)}$$

taking Ist root $x-3=0$, $x=3$
putup the value of $x=3$ in eqⁿ (ii)

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$17x - 1 = A_1(3x+1) + A_2(x-3)$

$17 \times 3 - 1 = A_1(3 \times 3 + 1) + A_2(3-3)$

$51 - 1 = A_1(10) + A_2(0)$

$50 = 10A_1$

$A_1 = 5$

taking IInd root $3x+1=0 \Rightarrow 3x=-1 \Rightarrow x = \frac{-1}{3}$

put up the value of $x = \frac{-1}{3}$ in eq^n (ii)

$17x - 1 = A_1(3x+1) + A_2(x-3)$

$17\left(\frac{-1}{3}\right) - 1 = A_1\left[3 \times \left(\frac{-1}{3}\right) + 1\right] + A_2\left[\frac{1}{3} - 3\right]$

$\frac{-17}{3} - 1 = A_1(-1+1) + A_2\left(\frac{-1-9}{3}\right)$

$\frac{-17}{3} - 1 = 0 + \left(\frac{-10}{3}\right)A_2$

$\frac{-17-3}{3} = \frac{-10}{3}A_2$

$10A_2 = 20$

$A_2 = 2$

Substitue the value of $A_1 = 5$, $A_2 = 2$ in eq^n (i)

$$\frac{17x-1}{(x-3)(3x+1)} = \frac{5}{x-3} + \frac{2}{3x+1}$$

(10) $\frac{x-13}{x^2+2x-3}$

$$\frac{x-13}{x^2+3x-1x-3} \Rightarrow \frac{x-13}{x(x+3)-1(x+3)} \Rightarrow \frac{x-13}{(x+3)(x-1)}$$

$$\frac{x-13}{(x+3)(x-1)} = \frac{A_1}{(x+3)} + \frac{A_2}{(x+1)} \text{ —— (i)}$$

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$$\frac{x-13}{(x+3)(x-1)} = \frac{A_1(x-1)+A_2(x+3)}{(x+3)(x-1)}$$

$$x-13 = A_1(x-1)+A_2(x+3) \quad \text{— (ii)}$$

taking root of 1st $x+3=0 \Rightarrow x=-3$
put up the value of $x=-3$ in eq^n - (ii)

$$x-13 = A_1(x-1)+A_2(x+3)$$
$$-3-13 = A_1(-3-1)+A_2(-3+3)$$
$$-16 = A_1(-4) + A_2(0)$$
$$+16 = +4A_1$$
$$A_1 = 4$$

taking root of IInd $x-1=0 \Rightarrow x=1$
put up the value of $x=1$ in eq^n -(ii)

$$x-13 = A_1(x-1)+A_2(x+3)$$
$$1-13 = A_1(1-1)+A_2(1+3)$$
$$-12 = 0+4A_2$$
$$A_2 = \frac{-12}{4} = -3$$

Substituting the value of $A_1 = 7$, $A_2 = -3$ in eq^n (i)

$$\frac{8x-10}{(2x+1)(2x-1)} = \frac{A_1}{(2x+1)} + \frac{A_2}{(2x-1)}$$

$$\frac{8x-10}{(2x+1)(2x-1)} = \frac{7}{(2x+1)} + \frac{-3}{(2x-1)}$$

$$\frac{8x-10}{(2x+1)(2x-1)} = \frac{7}{(2x+1)} - \frac{3}{(2x-1)}$$

(12) $\frac{3x-5}{(x-1)(x+2)}$

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$$\frac{3x-5}{(x-1)(x+2)} = \frac{A_1}{(x-1)} + \frac{A_2}{(x+2)} \quad \text{--- (i)}$$

$$\frac{3x-5}{(x-1)(x+2)} = \frac{A_1(x+2)+A_2(x-1)}{(x-1)(x+2)}$$

$$3x-5 = A_1(x+2) + A_2(x-1) \quad \text{--- (ii)}$$

taking Ist root of $x-1=0 \Rightarrow x=1$
put up the value of $x=1$ in eq^n (ii)

$$3x-5 = A_1(x+2) + A_2(x-1)$$
$$3(1)-5 = A_1(1+2) + A_2(1-1)$$
$$3-5 = A_1(3) + 0$$
$$-2 = 3A_1$$
$$A_1 = \frac{-2}{3}$$

taking IInd root of $x+2=0 \Rightarrow x=-2$
put up the value of $x=-2$ in eq^n (ii)

$$3x-5 = A_1(x+2) + A_2(x-1)$$
$$3(-2)-5 = A_1(-2+2) + A_2(-2-1)$$
$$-6-5 = A_1(0) + A_2(-3)$$
$$-11 = -3A_2$$
$$A_2 = \frac{11}{3}$$

Substituting the value of $A_1 = \frac{-2}{3}$, $A_2 = \frac{11}{3}$ in eq^n (i)

$$\frac{3x-5}{(x+2)(x-1)} = \frac{A_1}{(x-1)} + \frac{A_2}{(x+2)}$$

$$\frac{3x-5}{(x+2)(x-1)} = \frac{\frac{-2}{3}}{(x-1)} + \frac{\frac{11}{3}}{(x+2)}$$

$$\frac{3x-5}{(x+2)(x-1)} = \frac{\frac{-2x-4+11x-11}{3}}{(x-1)(x+2)}$$

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$$\frac{3x-5}{(x+2)(x-1)} = \frac{\frac{9x-15}{3}}{(x-1)(x+2)}$$

$$\frac{3x-5}{(x+2)(x-1)} = \frac{\frac{3(3x-5)}{3}}{(x-1)(x+2)}$$

$$\frac{3x-5}{(x+2)(x-1)} = \frac{3x-5}{(x-1)(x+2)}$$

13 $\frac{2x-3}{(x+5)(x-2)}$

$$\frac{2x-3}{(x+5)(x-2)} = \frac{A_1}{(x+5)} + \frac{A_2}{(x-2)} \quad -(i)$$

$$\frac{2x-3}{(x+5)(x-2)} = \frac{A_1(x-2)+A_2(x+5)}{(x+5)(x-2)}$$

$$\frac{2x-3}{(x+5)(x-2)} = \frac{A_1(x-2)+A_2(x+5)}{(x+5)(x-2)}$$

$$2x-3 = A_1(x-2) + A_2(x+5) \quad -(ii)$$

taking 1st root $x+5=0 \Rightarrow x=-5$
put the value of $x=-5$ in eq^n (ii)

$$2x-3 = A_1(x-2) + A_2(x+5)$$
$$2(-5)-3 = A_1(-5-2) + A_2(-5+5)$$
$$-10-3 = A_1(-7) + 0$$
$$-13 = -7A_1$$
$$A_1 = \frac{13}{7}$$

taking IInd root $x=2=0 \Rightarrow x=2$ put the value of $x=2$ in eq^n (ii)

$$2x-3 = A_1(x-2) + A_2(x+5)$$
$$2x-3 = A_1(2-2) + A_2(2+5)$$
$$2(2)-3 = 0 + A_2(7)$$

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$4 - 3 = 7A_2$

$1 = 7A_2 \Rightarrow A_2 = \frac{1}{7}$

Substituting the value of A_1 and A_2 in eqⁿ (i),

$$\frac{2x-3}{(x+5)(x-2)} = \frac{A_1}{(x+5)} + \frac{A_2}{(x-2)}$$

$$\frac{2x-3}{(x+5)(x-2)} = \frac{13/7}{(x+5)} + \frac{1/7}{(x-2)}$$

Polynomial

Analzebric expression of the form

$Fx = a_0x^n + a_1x^{n-1} + \ldots\ldots + a_n$ is called a polynomial in x, where $a_0 \neq 0, a_1, a_2 \ldots a_n$ are real constant and x is an unknown variable. The highest power of x that exists in the expression is called the degree of the polynomial.

For ex. $F(x) = x^5 + 4x^4 + 3x^3 + 4x^2 + x + 1$

$a_0 = 1$, $a_1 = 4$, $a_2 = 3$, $a_3 = 4$, $a_4 = 1$

$a_5 = 1$ are ....

Real constants and degree is 5, because the highest power of x is 5.

Rational Fraction: The quotient $\frac{P(x)}{Q(x)}$ of two

Polynomial $P(x)$ and $Q(x) \neq 0$ is called fraction explain-

$P(x) = x^2 + 3x + 5$

$Q(x) = 3x^2 + 4x^2 + 5x$

$$\frac{P(x)}{Q(x)} = \frac{x^2 + 3x + 5}{3x^2 + 4x^2 + 5x}$$