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LOGARITHMS

Definition :- "How many of one number do we multiply to get another number.

For ex → How many 2's do we multiply to get 8?

$8 = 2 \times 2 \times 2$

$8 = 2^3$

So, the logarithm of $\log_2(8)$ is 3.

Q. What is $\log_5(625) = ?$

$625 = 5 \times 5 \times 5 \times 5$

$625 = 5^4$

So, the logarithm of $\log_5 625$ is 4

Q. What is $\log_2(64) = ?$

$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$64 = 2^6$

So, the logarithm of $\log_2 64$ is 6

Q. What is $\log_{10}(1000) = ?$

$1000 = 10 \times 10 \times 10$

$1000 = 10^3$

So, the logarithm of $\log_{10}(1000)$ is 3.

Q. What is $\log_3(81) = ?$

$81 = 3 \times 3 \times 3 \times 3$

$81 = 3^4$

So, the logarithm of $\log_3(81)$ is 4.

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Q. What is $\log_2 (0.015625) = ?$

$$\frac{0.015625}{1000000} = \frac{3125}{100000} = \frac{625}{40000} = \frac{125}{8000} = \frac{25}{1600}$$

$$\frac{5}{320} = \frac{1}{64} = \frac{1}{2 \times 2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{2^6} = 2^{-6}$$

So, the logarithm of $\log_2 (0.015625)$ is 2^{-6}.

Q. What is $\log_4 (0.0625) = ?$

$$\frac{0.0625}{10000} = \frac{125}{20000} = \frac{25}{400} = \frac{5}{80} = \frac{1}{16}$$

$$\frac{1}{4 \times 4} = \frac{1}{4^2} = 4^{-2}$$

So, the logarithm of $\log_4 (0.0625)$ is -2

Q. What is $\log (0.0016) = ?$

$$\frac{0.0016}{10000} = \frac{2}{2500} = \frac{1}{1250}$$

$$\frac{1}{1250} = \frac{1}{5 \times 5 \times 5 \times 5}$$

$$= \frac{1}{5^4}$$

$$= 5^{-4}$$

So, the logarithm of $\log_5 (0.0016)$ is -4

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Q. What is $\log_8(0.125)$?

$$\frac{0.125}{1000} = \frac{1}{8} = \frac{1}{2\times2\times2} = \frac{1}{2^3} = 8^{-1}$$

So, the logarithm of $\log_8(0.125)$ is -1

Q. What is $\log_5(0.0008)$

$$\frac{0.0008}{10000} = \frac{1}{1250} = \frac{1}{5\times5\times5\times\cancel{5}} = \frac{1}{5^3} = 5^{-3}$$

So, the logarithm of $\log_5(0.008)$ is -3

Q. $\log_4(256)$

$$256 = 4\times4\times4\times4 = 4^4$$

So, the logarithm of $\log_4(256)$ is 4

Q. What is $\log_5(0.0016)$

$$\frac{0.0016}{10000} = \frac{1}{625} = \frac{1}{5\times5\times5\times5} = \frac{1}{5^4} = 5^{-4}$$

So, the logarithm of $\log_5(0.0016)$ is -4

Q What is $\log_3(\cancel{0.}729)$

$$729 = 3\times3\times3\times3\times3\times3 = 3^6$$

So, the logarithm of $\log_3(729)$ is 6

Q. What is $\log_3(2187)$

$$2187 = 3\times3\times3\times3\times3\times3\times3 = 3^7$$

So, the logarithm of $\log_3(2187)$ is 7

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Q. What is $\log_4 (0.0625)$

$\frac{0.0625}{10000} = \frac{1}{16} = \frac{1}{4\times4} = 4^{-2}$

So, the logarithm of $\log_4(0.0625)$ is -2

★ **Properties of Logarithm** -

1. $\log_a(m\times n) = \log_a(m) + \log_a(n)$ [The log of multiplication is the sum of the logs]
2. $\log_a\left(\frac{m}{n}\right) = \log_a(m) - \log_a(n)$ [The log of division is the difference of the logs]
3. $\log_a \frac{1}{n} = -\log_a(n)$ [This just follow or from the previous division rule]
4. $\log_a m^r = r \log_a(m)$ [The log of m with an exponent r is r times the log of m.]

Q. Simplify $\log_a[(x^2+1)^4 \cdot \sqrt{x}]$

$\log_a(x^2+1)^4 + \log_a \sqrt{x}$ [Using 1st property]

$4\log_a(x^2+1)^4 + \log_a \sqrt{x}$ [Using 4th property]

$4\log_a(x^2+1)^4 + \log_a x^{\frac{1}{2}}$

$4\log_a(x^2+1) + \frac{1}{2}\log_a x$ [Using 4th property]

Q. Simplify $\log_a(5) + \log_a(x) - \log_a(2)$

$\log_a(5\times x) - \log_a(2)$ [Using 1st property

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$\log_a \frac{5x}{2}$ By using 3rd property

COMMON LOGARITHM

Sometimes logarithm is written without a base like this
$\log(100)$

This actually means that the base is really 10.
It is called common logarithm
For example = $\log_{10}(1000) = 3$

CHARACTERSTICS AND MANTISSA

Characterstics of a logarithm is its integer part and Mantissa of that logarithm is to fractional part.

1. Positive integer

[1] ⇒	I.P.	F.P	characterstics - 1
	1	0	Fractional Part - 0

2. Positive integer with its Fractional Part

[1.2] ⇒	I.P	F.P	characterstics = 1
	1	.2	Mantissa = .2

3. Negative integer

[-1] ⇒	I.P	F.P	Characterstics = -1
	-1	0	Mantissa = 0

4. Negative integer with its Fractional Part
[-1.2]

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I.P	F.P		
-1	-.2	Characteristics	-2
-1+[-1]	1+[-.2]	Mantissa	.8
-2	.8		

Q. [-2.7]

I.P	F.P		
-2	-.7	Characteristics	-1
-2+[-1]	1+[-.7]	Mantissa	.3
-3	.3		

Q Find the characterstics and Mantissa of given number

① [-208.377]

I.P	F.P		
-208	-0.377	Characteristics	-209
-1+[-208]	1+[-0.377)	Mantissa	0.623
-1 - 208	0.623		
-209			

② [-438.732]

I.P	F.P		
-438	-.732		
-1+[-438]	1+[-.732]	Characteristic	-439
-1 - 438	1 - .732	Mantissa	.268
-439	0.268		

③ [-99.401]

I.P	F.P		
-99	-.401	Characteristic	-100
-1+[-99]	1+[-.401]	Mantissa	.599

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$-1-99$ $\qquad$ $1-.401$

-100 $\qquad$ 0.599

(4) $[-84.132]$

I.P	F.P
-84	$-.132$
$-1+[-84]$	$1+[-.132]$
$-1-84$	$1-.132$
-85	0.868

Characteristic -85
Mantissa $.868$

(5) $[-789.792]$

I.P	F.P
-789	$-.792$
$-1+[-789]$	$1+[-.792]$
$-1-789$	$1-.792$
-790	0.208

Characteristic -790
Mantissa $.208$

Q. Simplify $\log_6 24 + 2\log_6 3$.

$\log_6 24 + \log_6 3^2$ [using 4th property]

$\log_6 24 + \log_6 9$

$\log_6 (24 \times 9)$ [using 1st property]

$\log_6 216 = \log_6 (6 \times 6 \times 6)$

So the logarithm of $\log_6 216$ is 3

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Function

Definition :- A Function relates ~~&~~ an input to an output

input → $\boxed{\text{Relationship / Functions}}$ → output

It is like a machine that has an output ondaninput and the output is related somehow to the input

Example :- $F(x) = x^2 + 2x + 1$
input

$F(q) = q^2 + 2q + 1$
$F(z) = z^2 + 2z + 1$
$F(y) = y^2 + 2y + 1$

"$F(x) = ...$" is the classic way of writing a function

Ex - $F(x) = \frac{x}{2}$ "f of x equal to divided by 2."

1...100 it is a function because each input x has a single output $\frac{x}{2}$

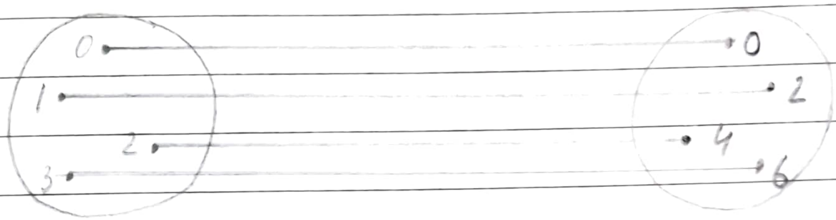
Ex - "Multiply by 2" is a very simple Function here are the three parts

Input	Relationship / Function	Output
$x = 0$ to 5	$f(x) = 2x$	
0	2×0	0
1	2×1	2
2	2×2	4

3	2×3	6
4	2×4	8
5	2×5	10

1. One to one (This is ok in function)

input output

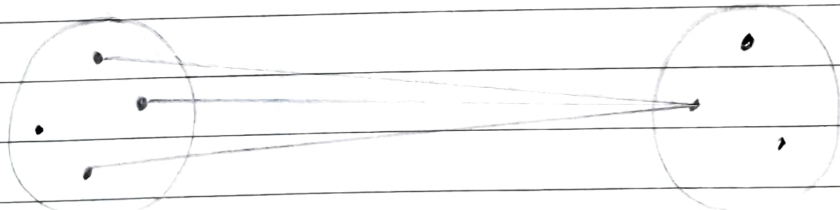


2. One to many (This is not possible)

input output

3. Many to one (This is ok in function)

input output



-4 -8 -16
-12 -16
-28
-2 -4 -4
-6 -4

Ex Show that the equation $F(x) = x + 2x - x^2$ is a function or not on the given input 1 to 5

Input	Function	Output
$x = 0$ to 5	$F(x) = x + 2x + x^2$	
1	$1 + 2 \times 1 - (1)^2$	2

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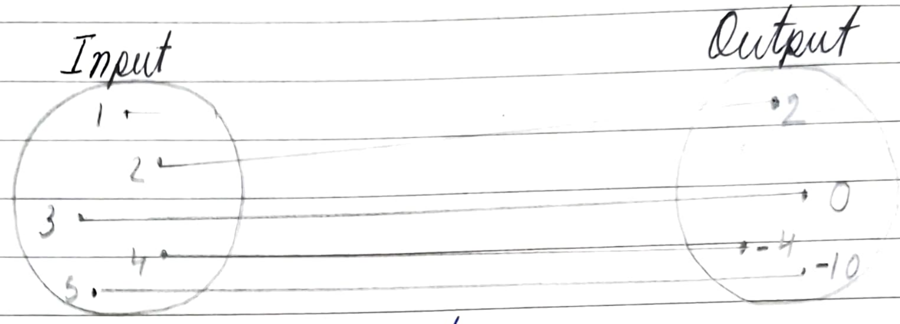
2	$2+2\times2-(2)^2$	2
3	$3+2\times3-(3)^2$	0
4	$4+2\times4-(4)^2$	-4
5	$5+2\times5-(5)^2$	-10

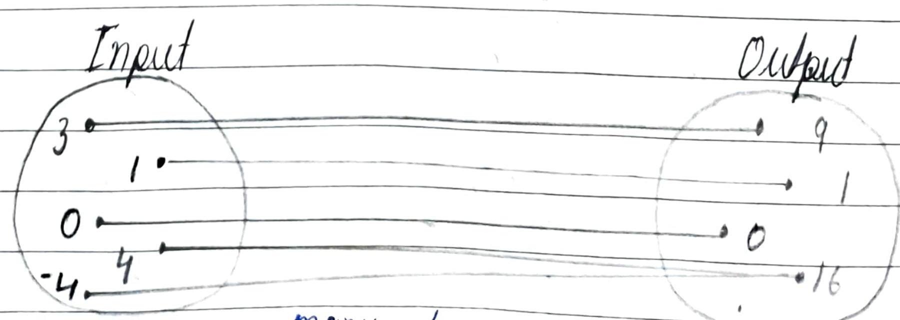


many to one

here Follow the condition many to one hence it is proved that $F(x)=x+2x-x^2$ is a function on the given input 1 to 5.

Q Check the relationships $F(x)=x^2$ is a function or not on the given input 3, 1, 0, 4, -4

Input	Function	Output
$x=3,1,0,4-4$	$F(x)=x^2$	
3	$(3)^2$	9
1	$(1)^2$	1
0	$(0)^2$	0
4	$(4)^2$	16
-4	$(-4)^2$	16



many to one

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here follow the condition many to one hence it is proved that $F(x) = x^2$ is a function on the given input 3, 1, 0, 4, -4

Q. Check the equation $F(z) = 2 + z - z^2$ is a function or not on the given input -3 to +3

Input	Function	Output
$x = -3, -2, -1, 0, 1, 2, 3$	$F(z) = 2 + z - z^2$	
-3	$2 + (-3) - (-3)^2$	8
-2	$2 + (-2) - (-2)^2$	4
-1	$2 + (-1) - (-1)^2$	2
0	$2 + 0 - (0)^2$	2
1	$2 + 1 - (1)^2$	2
2	$2 + 2 - (2)^2$	0
3	$2 + 3 - (3)^2$	-4

Many to one

here follow the condition many to one hence it is proof that $F(z) = 2 + z + z^2$ is a function on the given input -3 to +3